Closing Wed: HW_3A, 3B, 3C

Midterm 1 is Thurs: 4.9, 5.1-5.5, 6.1-6.3

Entry Task:

Let R be the region bounded by

$$y = \frac{1}{x^2} + \frac{1}{x}, y = 0, x = 1, x = 2.$$

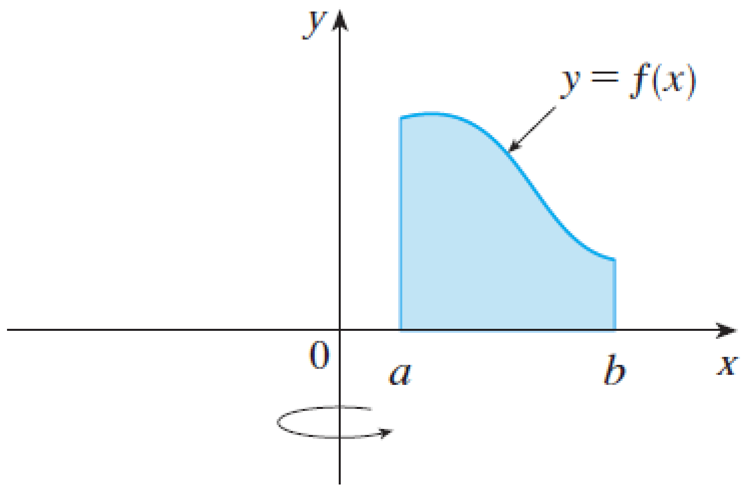
(a) Set up an integral for the volume of the solid obtained by rotating R about the x-axis.

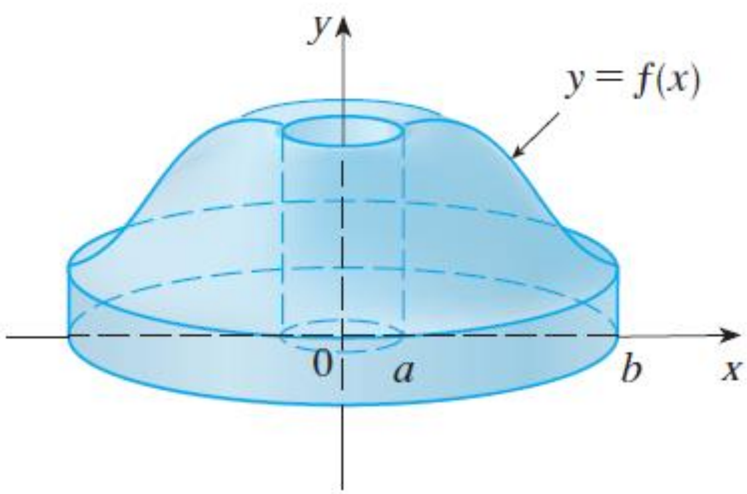
(b) Try to use cross-sectional slicing to set up an integral for the volume obtained by rotating R about the *y*-axis. Why is this difficult/messy?

6.3 Volumes Using Cylindrical Shells

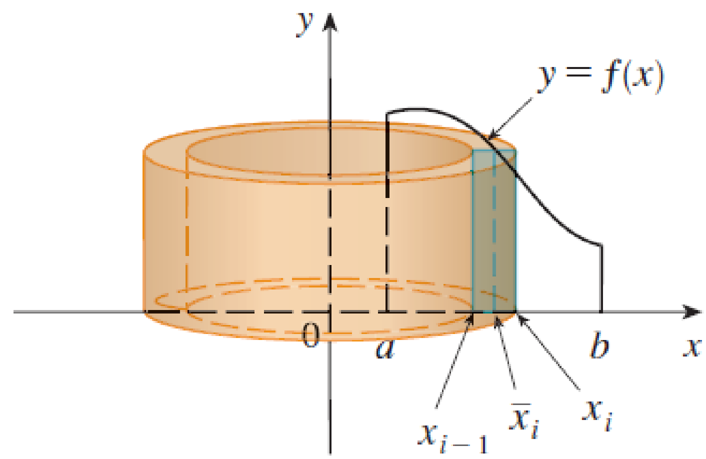
Visual Motivation:

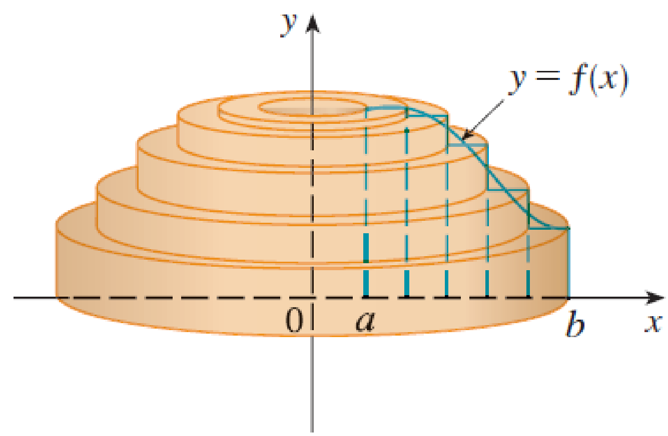
Consider the solid





We want to us "dx", but that breaking the region into thin vertical subdivisions and rotating those gives a new shape, "cylindrical shells"





Derivation:

The pattern for the volume of one thin cylindrical shell is

VOLUME = 2π(radius)(height)(thickness)

= (surface area)(thickness)

Thus, if we can find a formula, $SA(x_i)$, for the surface area of a typical cylindrical shell, then

Thin Shell Volume ≈ $SA(x_i)\,\Delta x$,

$$\text{Total Volume} \approx \sum_{i=1}^{n} \mathrm{SA}(\mathrm{x}_i)\ \Delta\mathrm{x}$$

$$\text{Exact Volume} = \lim_{n\to\infty} \sum_{i=1}^{n} \mathrm{SA}(\mathrm{x}_i)\Delta\mathrm{x}$$

$$\text{Volume} = \int_a^b SA(x)dx$$

$$= \int_a^b 2\pi(radius)(height)dx$$

Closing Wed: HW_3A, 3B, 3C
Midterm 1 is Thurs: 4.9, 5.1-5.5, 6.1-6.3

Entry Task:
Let R be the region bounded by

$$y = x^3, y = 4x,$$

between x = 1 and $x = 2$.

1. Set up the integrals for the volume of the solid obtained by rotating R **about the y-axis**.
   (a) Using dy.
   (b) Using dx.

2. What changes if we rotate about the vertical line x = -2?

3. What changes if we rotate about the vertical line x = 3?

Volume using cylindrical shells

1. Draw a typical rectangle **parallel** to the axis of rotation and draw a typical cylindrical shell.
   Label the thickness dx or dy appropriately.
   Label everything in terms of this variable.

2. Find the formula for the surface area of a typical shell:
   radius = ? (looks like *x*, *x*-*a* or *a*-*x*)
   height = ? (involves the functions)

3. Integrate!

$$\int_a^b 2\pi(\text{radius})(\text{height})(\text{dx or dy})$$

Volume using cross-sectional slicing

1. Draw a typical rectangle **perpendicular** to the axis of rotation and cross-section.
   Label the thickness dx or dy appropriately.
   Label everything in terms of this variable.

2. Find the formula for the cross-sectional area:

Disc: Area = $\pi(\text{radius})^2$

Washer: Area = $\pi(\text{outer})^2 - \pi(\text{inner})^2$

3. Integrate!

$$\int_a^b (\pi(\text{outer})^2 - \pi(\text{inner})^2)(\text{dx or dy})$$

Summary of Volume Methods

Step 1: Choose the variable you want to use
(based on the region and the given equations)

Step 2: Draw typical rectangle based on the variable you chose which will either be perpendicular (disc/washer) or parallel (shells) to the axis of rotation.
Draw a typical disc/washer or shell.
Label thickness as dx or dy appropriately.
Label everything else in terms of this variable.

Step 3: *Cross-sections*: Find pattern for radius of disc/washers.
Shells: Find pattern for radius and height of shells.

Step 4: Integrate the appropriate pattern as we have discussed.

If you still are having trouble seeing which variable goes with which method here is a summary:

Axis of rotation	**Disc/Washer**	**Shells**
x-axis (or any horizontal axis)	dx	dy
y-axis (or any vertical axis)	dy	dx